

## Entering AP Calculus AB Summer Review Assignment

Welcome to AP Calculus AB!!! I am so excited to spend our year together continuing our journey through mathematics.

Directions.

- ♥ This assignment will be collected on the 1<sup>st</sup> day of classes. Exact Date TBD.
- ♥ Work must be organized and labeled. You may work on this document or you may use your own paper. Organized work is very important on the AP Exam given each May. Please take extra care to make sure that your work is easy to follow and your answers are easy to read.
- ♥ This assignment will be graded for accuracy and will count as your first quiz grade for Quarter 1.
- ♥ I do respond to email over the summer!!! If you every have a question or need a hint or two...please reach out.....I will respond in a timely manner.

I am really looking forward to working together next school year.....have a great summer.

Be safe and stay well.



Mrs. Weber

Topic A: Functions

1.) If  $f(x) = 4x - x^2$ , find:

a.)  $f(4) - f(-4)$

b.)  $\sqrt{f\left(\frac{3}{2}\right)}$

c.)  $\frac{f(x+h) - f(x)}{2h}$

2.) If  $V(r) = \frac{4}{3}\pi r^3$ , find:

a.)  $V\left(\frac{3}{4}\right)$

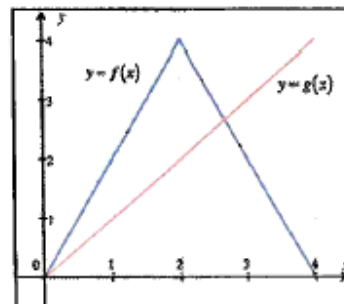
b.)  $V(r+1) - V(r-1)$

c.)  $\frac{V(2r)}{V(r)}$

3.) If  $f(x)$  and  $g(x)$  are given in the graph, find:

a.)  $(f - g)(3)$

b.)  $f(g(3))$



4.) If  $f(x) = \begin{cases} -x, & x < 0 \\ x^2 - 1, & 0 \leq x < 2, \\ \sqrt{x+2} - 2, & x \geq 2 \end{cases}$  find:

a.)  $f(0) - f(2)$

b.)  $\sqrt{5 - f(-4)}$

c.)  $f(f(3))$

## Topic B: Domain and Range

Find the domain of the following functions using interval notation:

1.)  $f(x) = 3$

2.)  $y = x^3 - x^2 + x$

3.)  $y = \frac{x^3 - x^2 + x}{x}$

4.)  $y = \frac{x-4}{x^2-16}$

5.)  $f(x) = \frac{1}{4x^2 - 4x - 3}$

6.)  $y = \sqrt{2x-9}$

7.)  $y = \log(x-10)$

8.)  $y = \frac{\sqrt{2x+14}}{x^2-49}$

Find the range of the following functions:

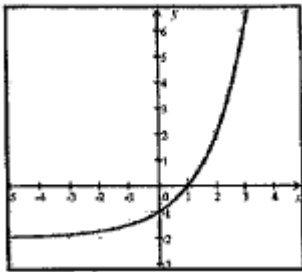
9.)  $y = x^4 + x^2 - 1$

10.)  $y = 100^x$

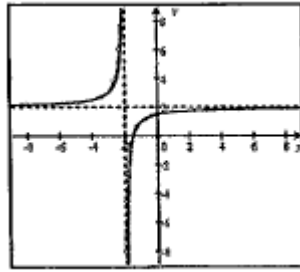
11.)  $y = \sqrt{x^2 + 1} + 1$

Find the domain and range of the following functions using interval notation.

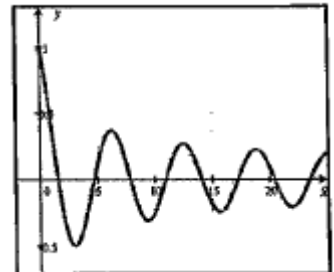
12.)



13.)



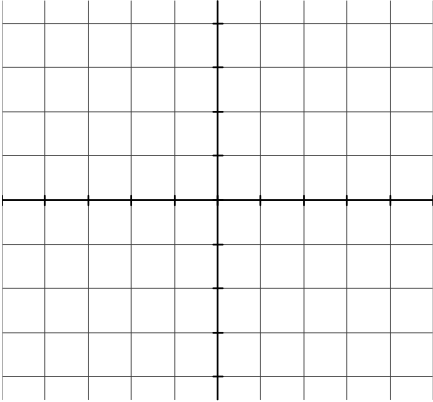
14.)



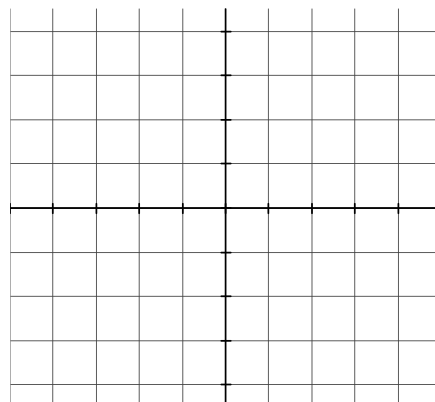
## Topic C: Graphs of Common Functions

Sketch each of the following as accurately as possible. **You will need to be VERY familiar with each of these graphs throughout the year.** You may use a graphing calculator for some of them if you have access to one over the summer. Again, these are VERY important graphs to know. Be very accurate with regards to “open circles” and “closed circles” as those features may not be revealed on a graphing utility.

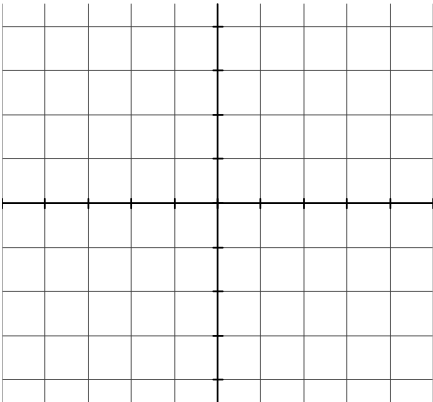
1.  $y = x$



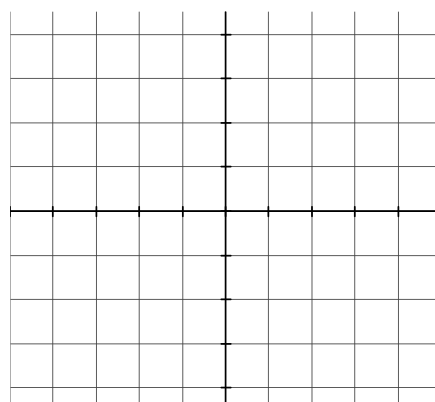
2.  $y = x^2$



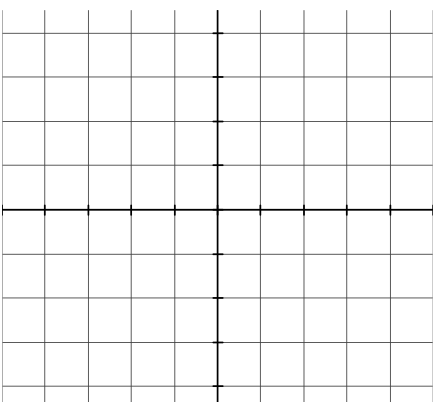
3.  $y = x^3$



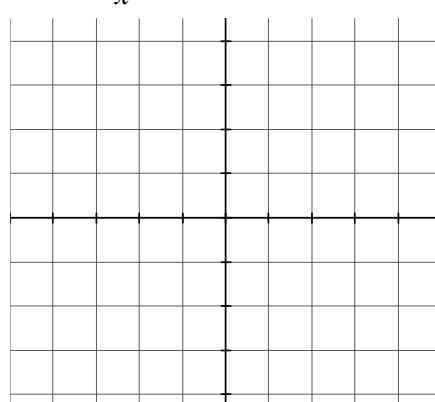
4.  $y = \sqrt{x}$



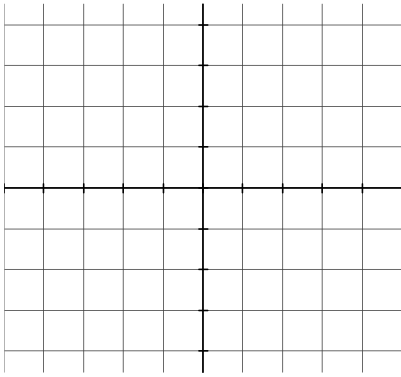
5.  $y = |x|$



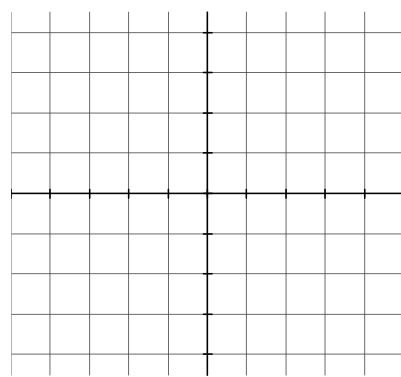
6.  $y = \frac{|x|}{x}$



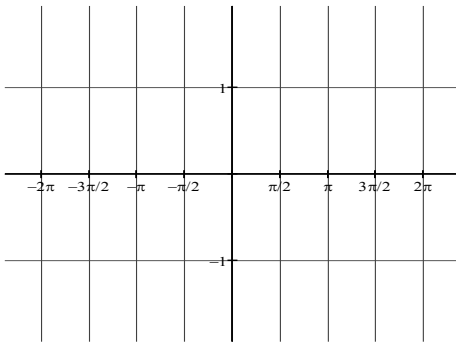
7.  $y = x^{1/3}$



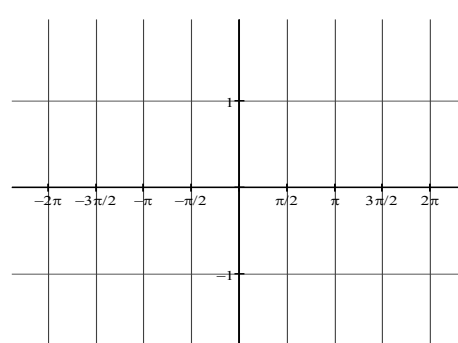
8.  $y = x^{2/3}$



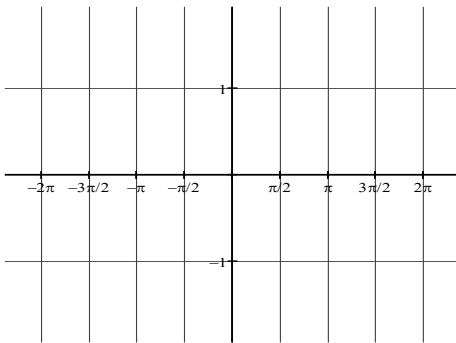
9.  $y = \sin x$



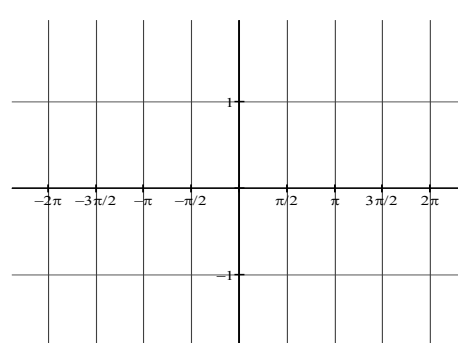
10.  $y = \cos x$



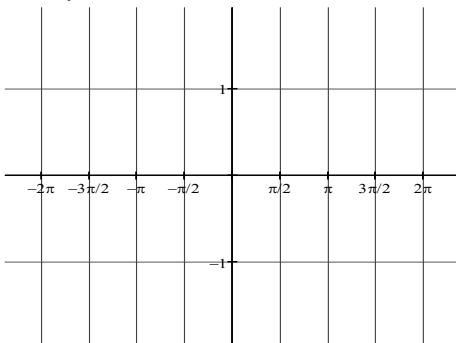
11.  $y = \tan x$



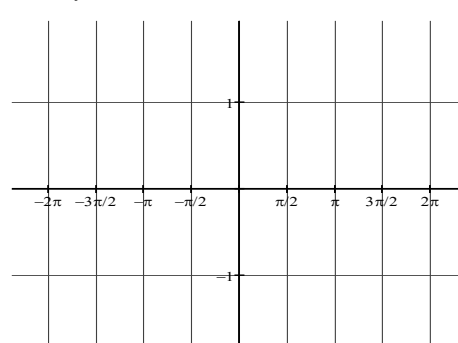
12.  $y = \cot x$



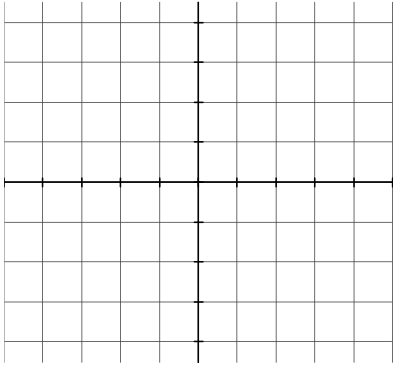
13.  $y = \sec x$



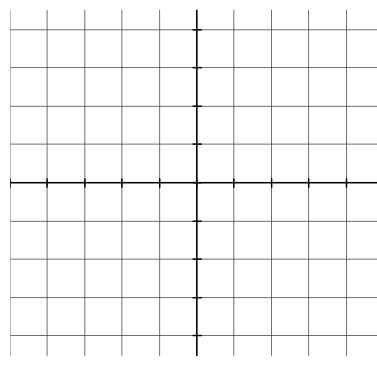
14.  $y = \csc x$



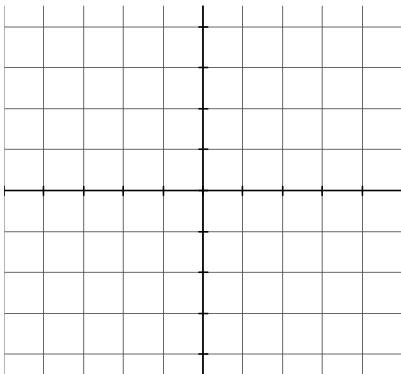
15.  $y = e^x$



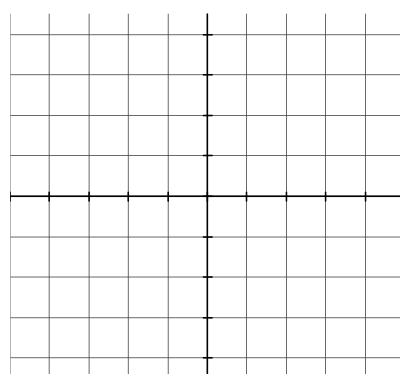
16.  $y = \ln x$



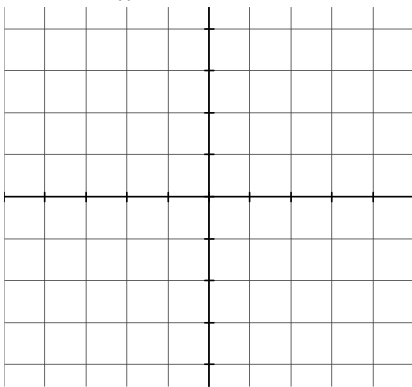
17.  $y = \frac{1}{x}$



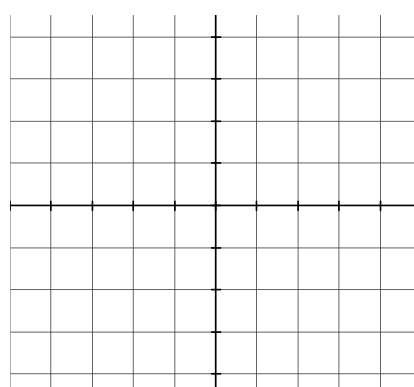
18.  $y = \llbracket x \rrbracket$



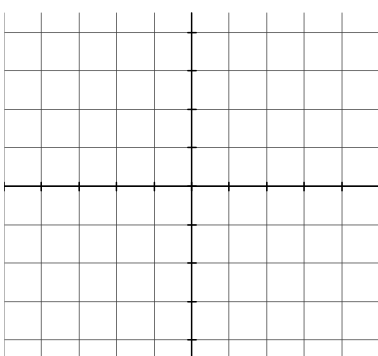
19.  $y = \frac{1}{x^2}$



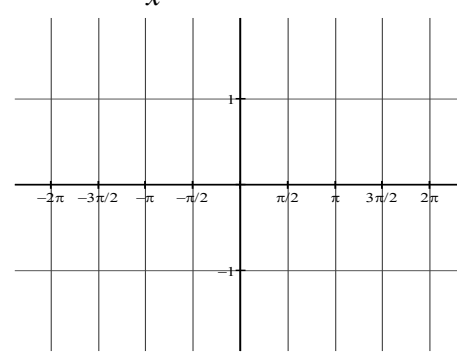
20.  $y = 2^x$



21.  $y = \sqrt{4 - x^2}$



22.  $y = \frac{\sin x}{x}$



### Topic D: Even/Odd Functions and Symmetry

Show work to determine if the relation is even, odd, or neither. You may want to research how to determine evenness and oddness.

1.)  $f(x) = 7$

2.)  $f(x) = 2x^2 - 4x$

3.)  $f(x) = -3x^3 - 2x$

4.)  $f(x) = \sqrt{x+1}$

5.)  $f(x) = \sqrt{x^2 + 1}$

6.)  $f(x) = |8x|$

Show work to determine if the graphs of these equations are symmetric to the  $x$ -axis,  $y$ -axis, or the origin.

7.)  $4x = 1$

8.)  $y^2 = 2x^4 + 6$

9.)  $3x^2 = 4y^3$

10.)  $x = |y|$

11.)  $|x| = |y|$

12.)  $|x| = y^2 + 2y + 1$

## Topic E: Function Transformations

If  $f(x) = x^2 - 1$ , describe in words what the following would do to the graph of  $f(x)$ :

1.)  $f(x) - 4$

2.)  $f(x - 4)$

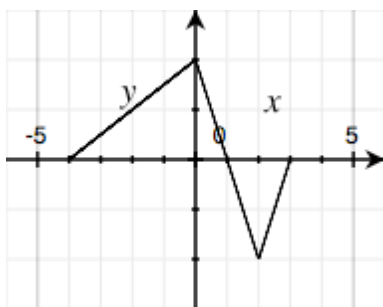
3.)  $-f(x + 2)$

4.)  $5f(x) + 3$

5.)  $f(2x)$

6.)  $|f(x)|$

Here is a graph of  $y = f(x)$ :

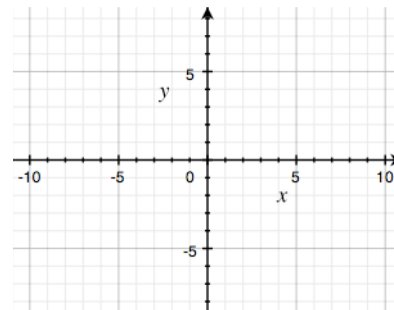
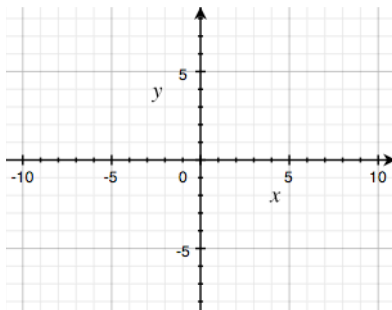
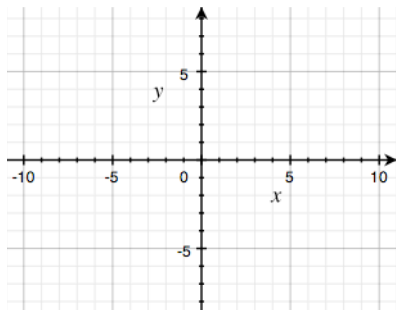


Sketch the following graphs:

7.)  $y = 2f(x)$

8.)  $y = -f(x)$

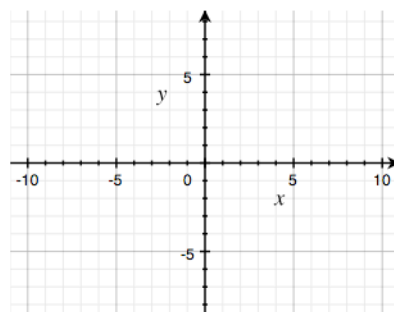
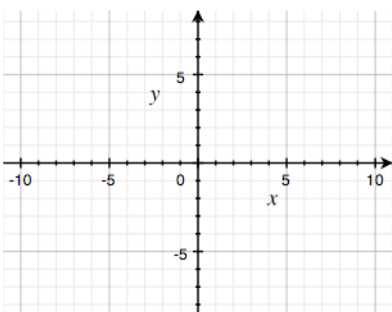
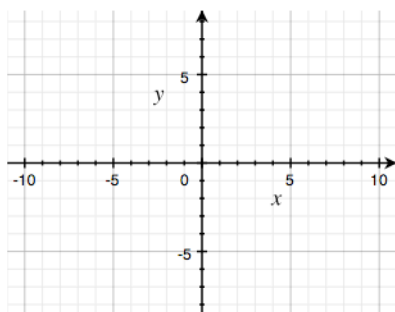
9.)  $y = f(x - 1)$



10.)  $y = f(x) + 2$

11.)  $y = |f(x)|$

12.)  $y = f(|x|)$





## Topic F: Special Factorization

Factor completely.

1.)  $x^3 + 8$

2.)  $x^3 - 8$

3.)  $27x^3 - 125y^3$

4.)  $x^4 + 11x^2 - 80$

5.)  $ac + cd - ab - bd$

6.)  $2x^2 + 50y^2 - 20xy$

7.)  $x^2 + 12x + 36 - 9y^2$

8.)  $x^3 - xy^2 + x^2y - y^3$

9.)  $(x-3)^2(2x+1)^3 + (x-3)^3(2x+1)^2$

## Topic G: Linear Functions

1.) Find the equation of the line in point-slope form, with the given slope, passing through the given point.

a.)  $m = -7$ ,  $(-3, -7)$

b.)  $m = -\frac{1}{2}$ ,  $(2, -8)$

c.)  $m = \frac{2}{3}$ ,  $\left(-6, \frac{1}{3}\right)$

2.) Find the equation of the line in point-slope form, passing through the given points.

a.)  $(-3, 6)$ ,  $(-1, 2)$

b.)  $(-7, 1)$ ,  $(3, -4)$

c.)  $\left(-2, \frac{2}{3}\right)$ ,  $\left(\frac{1}{2}, 1\right)$

3.) Find the equations of the lines through the given point that are a.) parallel and b.) normal to the given line.

a.)  $(5, -3)$ ,  $x + y = 4$

b.)  $(-6, 2)$ ,  $5x + 2y = 7$

c.)  $(-3, -4)$ ,  $y = -2$

4.) Find the equation of the line in general form, containing the point  $(4, -2)$  and parallel to the line containing the points  $(-1, 4)$  and  $(2, 3)$ .

5.) Find  $k$  if the lines  $3x - 5y = 9$  and  $2x + ky = 11$  are a.) parallel and b.) perpendicular.

## Topic H: Solving Quadratic and Polynomial Equations

Solve each equation for  $x$  over the real number system.

1.)  $x^2 + 7x - 18 = 0$

2.)  $x^2 + x + \frac{1}{4} = 0$

3.)  $2x^2 - 72 = 0$

4.)  $12x^2 - 5x = 2$

5.)  $20x^2 - 56x + 15 = 0$

6.)  $81x^2 + 72x + 16 = 0$

7.)  $x + \frac{1}{x} = \frac{17}{4}$

8.)  $x^3 - 5x^2 + 5x - 25 = 0$

9.)  $2x^4 - 15x^3 + 18x^2 = 0$

10.) If  $y = x^2 + kx - k$ , for what values of  $k$  will the quadratic have two real solutions?

## Topic I: Asymptotes

For each function, find the equations of both the vertical asymptote(s) and horizontal asymptote (if it exists) and the location of any holes.

$$1.) y = \frac{x-1}{x+5}$$

$$2.) y = \frac{8}{x^2}$$

$$3.) y = \frac{2x+16}{x+8}$$

$$4.) y = \frac{2x^2+6x}{x^2+5x+6}$$

$$5.) y = \frac{x}{x^2-25}$$

$$6.) y = \frac{x^2-5}{2x^2-12}$$

$$7.) y = \frac{x^3}{x^2+4}$$

$$8.) y = \frac{x^3+4x}{x^3-2x^2+4x-8}$$

$$9.) y = \frac{10x+20}{x^3-2x^2-4x+8}$$

$$10.) y = \frac{1}{x} - \frac{x}{x+2} \text{ (Hint: Express with a common denominator)}$$

## Topic J: Negative and Fractional Exponents

Simplify and write with positive exponents.

1.)  $-12^2 x^{-5}$

2.)  $(-12x^5)^{-2}$

3.)  $(4x^{-1})^{-1}$

4.)  $\left(\frac{-4}{x^4}\right)^{-3}$

5.)  $\left(\frac{5x^3}{y^2}\right)^{-3}$

6.)  $(x^3 - 1)^{-2}$

7.)  $(121x^8)^{1/2}$

8.)  $(8x^2)^{-4/3}$

9.)  $(-32x^{-5})^{-3/5}$

10.)  $\frac{1}{4}(16x^2)^{-3/4}(32x)$

11.)  $\frac{(x^2 - 1)^{-1/2}}{(x^2 + 1)^{1/2}}$

12.)  $(x^{-2} + 2^{-2})^{-1}$

## Topic K: Complex Fractions

Eliminate the complex fractions:

$$1.) \frac{\frac{5}{8}}{\frac{2}{-\frac{3}{3}}}$$

$$2.) \frac{4 - \frac{2}{9}}{3 + \frac{4}{3}}$$

$$3.) \frac{2 + \frac{7}{2} + \frac{3}{5}}{5 - \frac{3}{4}}$$

$$4.) \frac{x - \frac{1}{x}}{x + \frac{1}{x}}$$

$$5.) \frac{1 + x^{-1}}{1 - x^{-2}}$$

$$6.) \frac{x^{-1} + y^{-1}}{x + y}$$

$$7.) \frac{x^{-2} + x^{-1} + 1}{x^{-2} - x}$$

$$8.) \frac{\frac{1}{3}(3x-4)^{-3/4}}{-\frac{3}{4}}$$

$$9.) \frac{2x(2x-1)^{1/2} - 2x^2(2x-1)^{-1/2}}{(2x-1)}$$

## Topic L: Inverses

Find the inverse of each of the following functions and use a graphing utility to show graphically that its inverse is a function. **No need to sketch the graph of the inverse. This is just so that you can verify your equations are truly inverses of each other!**

1.)  $2x - 6y = 1$

2.)  $y = ax + b$

3.)  $y = 9 - x^2, x \geq 0$

4.)  $y = \sqrt{1 - x^3}$

5.)  $y = \frac{9}{x}$

6.)  $y = \frac{2x + 1}{3 - 2x}$

Find the inverse of each of the following functions and show that  $f(f^{-1}(x)) = x$

7.)  $f(x) = \frac{1}{2}x - \frac{4}{5}$

8.)  $f(x) = x^2 - 4$

9.)  $f(x) = \frac{x^2}{x^2 + 1}$

10.) Without finding the inverse, find the domain and range of the inverse to  $f(x) = \frac{\sqrt{x+1}}{x^2}$

## Topic M: Adding Fractions and Solving Rational Equations

1.) Combine the following fractions:

a.)  $\frac{2}{3} - \frac{1}{x}$

b.)  $\frac{1}{x-3} + \frac{1}{x+3}$

c.)  $\frac{5}{2x} - \frac{5}{3x+15}$

d.)  $\frac{2x-1}{x-1} - \frac{3x}{2x+1}$

2.) Solve the equation for  $x$ .

a.)  $\frac{2}{3} - \frac{1}{x} = \frac{5}{6}$

b.)  $\frac{1}{x-3} + \frac{1}{x+3} = \frac{10}{x^2-9}$

c.)  $\frac{5}{2x} - \frac{5}{3(x+5)} = \frac{5}{x}$

d.)  $\frac{2x-1}{x-1} - \frac{3x}{2x+1} = \frac{x^2+11}{2x^2-x-1}$



## Topic N: Exponential Functions and Logarithms

Simplify the following:

1.)  $\log_2 \frac{1}{4}$

2.)  $\log_8 4$

3.)  $\ln \frac{1}{\sqrt[3]{e^2}}$

4.)  $5^{\log_5 40}$

5.)  $e^{\ln 12}$

6.)  $\log_{12} 2 + \log_{12} 9 + \log_{12} 8$

7.)  $\log_2 \frac{2}{3} + \log_2 \frac{3}{32}$

8.)  $\log_{\frac{1}{3}} \frac{4}{3} - \log_{\frac{1}{3}} 12$

9.)  $\log_3 (\sqrt{3})^5$

Solve the following:

10.)  $\log_5 (3x-8) = 2$

11.)  $\log_9 (x^2 - x + 3) = \frac{1}{2}$

12.)  $\log(x-3) + \log 5 = 2$

13.)  $\log_2 (x-1) + \log_2 (x+3) = 5$

14.)  $\log_5 (x+3) - \log_5 x = 2$

15.)  $\ln x^3 - \ln x^2 = \frac{1}{2}$

16.)  $3^{x-2} = 18$

17.)  $e^{3x+1} = 10$

18.)  $8^x = 5^{2x-1}$

## Topic O: Basic Right Angle Trigonometry

Solve the following:

If point  $P$  is on the terminal side of  $\theta$ , find all 6 trigonometric functions of  $\theta$ . (Answers need not be rationalized.)

1.)  $P(-2, 4)$

2.)  $P(\sqrt{5}, -2)$

3.) If  $\cos \theta = -\frac{5}{13}$ , in quadrant II,  
find  $\sin \theta$  and  $\tan \theta$ .

4.) If  $\cot \theta = \frac{2\sqrt{10}}{3}$ , in quadrant III,  
find  $\sin \theta$  and  $\cos \theta$ .

5.) State the quadrant in which each of the following is true.

a.)  $\sin \theta > 0$  and  $\cos \theta < 0$

b.)  $\csc \theta < 0$  and  $\cot \theta > 0$

c.)  $\tan \theta > 0$  and  $\sec \theta < 0$

## Topic P: Trigonometric Identities

Verify the following identities:

1.)  $(1 + \sin x)(1 - \sin x) = \cos^2 x$

2.)  $\sec^2 x + 3 = \tan^2 x + 4$

3.)  $\frac{1 - \sec x}{1 - \cos x} = -\sec x$

4.)  $\frac{1}{1 + \tan x} + \frac{1}{1 + \cot x} = 1$

5.)  $\csc(2x) = \frac{\csc x}{2 \cos x}$

6.)  $\frac{\cos(3x)}{\cos x} = 1 - 4 \sin^2 x$

### Topic Q: Solving Trigonometric Equations

Solve each equation on the interval  $[0, 2\pi)$ . Do not use a calculator.

1.)  $\sin^2 x = \sin x$

2.)  $3 \tan^3 x = \tan x$

3.)  $\sin^2 x = 3 \cos^2 x$

4.)  $\cos x + \sin x \tan x = 2$

5.)  $\sin x = \cos x$

6.)  $2 \cos^2 x + \sin x - 1 = 0$

### Topic R: Using the Graphing Calculator

Use your Graphing Calculator to solve each equation below. Problems (1 – 3)

1.)  $3x^3 - x - 5 = 0$

2.)  $2x^2 - 1 = 2^x$

3.)  $2\ln(x+1) = 5\cos x$  on  $[0, 2\pi)$

Use your graphing Calculator to find the solution (intersection) of the given system of equations.

4.) 
$$\begin{cases} f(x) = x^4 - 6.5x^2 + 6x + 2 \\ g(x) = 1 + x + e^{x^2 - 2x} \end{cases}$$

Use your graphing Calculator to find both a relative maximum and a relative minimum point of the given function.

5.)  $h(x) = 2x^5 - 3x^4 + x - 4$